Paper 1 - Algebra I

Group Theory : Review of basic notions; isomorphism theorems, automorphism, direct products, conjugacy classes, centraliser, normaliser, center. Structure Theorem for finite abelian groups.

Ring Theory : Definition and examples of rings and fields; ideals prime ideals, maximal ideals; homomorphism theorems and quotient rings; the field of quotients of an integral domain; units in Z/nZ.

Euclidean rings; polynomial rings; Principal ideal domain and unique factorisation domains, examples of imaginary extensions of Z

Linear Algebra : Recap definition of vector spaces, subspaces, bases, isomorphism theorems; direct summand of a vector space, direct sum of vector spaces; every subspace is a direct summand;

Inner product, (scalar product) space, orthogonal bases, Gram-Schmidt process; The dual space, dual basis, reflexivity of finite dimensional vector spaces,

Bilinear maps, multilinear maps, alternating n-forms, definition and properties of determinants. Bilinear forms, quadratic forms; Sylvester�s theorem.

Adjoints, symmetric operators, Hermitian operators, unitary operators;

Eigen values, eigen vectors, minimal and characteristic polynomials, Cayley-Hamilton theorem, Statement of the existence and uniqueness of Jordan canonical forms, computation of Jordan canonical form for three by three matrices, triangulation, diagonalisation of unitary maps. Invariant subspaces, Eigenvectors of symmetric linear maps, the spectral the or emhermitian case and symmetric case. Normal operators. Applications.

References :

1. Gallian, J.A. : Contemporary Abstract Algebra, (Fourth Ed.) Narosa, 1999.

2. Hoffman K, and Kunze R. : Linear Algebra (IInd Ed.) Prentice Hall of India, 1998.

3. Artin, M. : Algebra, Prentice Hall of India, 1994.

4. Lang S. : Linear Algebra, (IInd Ed.) Addison - Wesley, 1971.

5. Bhattacharya P.B., Jain S.K., Nagpaul : Basic Abstract Algebra (IInd Ed.) (Sp. Ed. For South Asia), Cambridge Univ. Press 1995.

6. Sahai V, Bist V.: Linear Algebra, Narosa Pub. House, 2002.

7. Dummit D.S., Foote R.M., : Abstract Algebra, J. Wiley (Indian Ed.) 2002.

8. Kumaresan, S., Linear Algebra � A Geometric Approach, Prentice Hall of India, 2000.

Paper 2 � Analysis I

The real Number System : Review of real number system, complete ordered field, topology of R, topology of Rn

,RLimited in R : Review of limits in convergence of sequences and series, rearrangements, conditional convergence.

n : Convergence of sequences andRLimits in series in Rn, compactness in Rn.

Continuity in Rn Continuity, uniform continuity, connectedness in Rn.

Monotone functions, functions of bounded , discontinuity.�variation in

Sequences and series of functions in Rn : Uniform convergence, Weirstrass M-test.

Differentiation : Basic notions in one variable case, definition for several variables, directional derivative, partial, total derivative, Jacobian matrix, Chain Rule, Sufficient condition for differentiability, Mean Value Theorem, Taylor formula for real valued functions of a vector variable, maxima and minima, inverse function theorem, implicit function theorem, maxima and minima with constraints, construction of smooth function with compact support.

Riemann Integration : Basic notions of Riemann Integrals in Rn, Iterated integrals, Fubini theorem, Leibnitz rule of differentiation under the integral sign. Improper integrals in R.

References :

1. Apostol T.: Mathematical Analysis, Second Edition, Addision Wesley, Narosa, (Indian Student Edition)

2. Spivak M.: Calculus on Manifolds, Benjamin / Commings, 1965

3. Rudin W.: Principles of math. Analysis (International Student Edition), Third Edition, McGraw-Hill / Kogakusha 1976.

4. Mankres J.R.: Analysis on Manifolds, Addison - Wesley Publishing Company, 1990.

5. Hevitt E, Stromberg K.: Real and Abstract Analysis, Springer � Narosa Publishing House, (Indian Print) 1978.

6. Bartle & Sherbert : Introduction to Real Analysis, 3rd Ed., John Wiley & Sons, (Asia), 2000.

7. Aliprantis C.D. & O. Burkinshaw : Principles of Real Analysis, Third Ed. Harcourt Asia Pte Ltd., 2002.

Paper 3 � Topology

Basic Set Theory : Sets and relations Partial orders, Equivalence relations, Zorn�s lemma, Axiom of Choice, Schroder - n etc.�, �, �, Z, Q, �Bernstein theorem, Cardinality of

General Notions of topology : Topological spaces, Bases and sub-bases, Continuous maps and equivalent versions, Open maps, Closed maps, Homeomophisms, countability axioms, T0,T1and T2 - spaces; Metric n as examples, Completeness, Baire�spaces, selected topological subspaces of category theorem.

Product Topology :

Connected and Compact spaces : Connected spaces, Path connected spaces, Compact spaces, Equivalent characterizations of Compact metric spaces, Continuous images of connected (respectively, compact) spaces; local compactness.

Quotient Topology with concrete examples with Sn as the quotient of the closed disc Dn and and Pn (R).

Homotopy and Fundamental Group : Path Homotopy, First Fundamental Group of a topological Space, Covering Spaces, Path-lifting theorem, Homotopy - lifting theorem, Computations of first fundamental groups of S1 and S2 .

References :

1. Munkres J.R.: Topology � Second Edition, Prentice Hall of India, 2002.

2. Simmons G.F.: Introduction to Topology and Modern Analysis, McGraw-Hill Book Company (International Edition)

3. Armstrong M.A.: Basic Topology, Springer-Verlag, 1983.

Paper 4 � Complex Analysis

Complex Number System : The field of complex numbers; polar form of complex numbers; extended complex plane; the point at infinity; stereo graphic projection; linear and Mobius transformations.

Sequences and series : Sequences and series; power series, radius of convergence, functions defined by power series, continuity of a power series map, algebra of power series maps; exponential, trigonometric and hyperbolic functions and their properties, branches of logarithm function, its properties.

Differentiation: Holomorphic functions, Cauchy-Riemann equations, term by term differentiability of a power series map, inverse function theorem, harmonic functions.

Complex Integration: Complex line integrals; integration along piece-wise smooth paths.

Cauchy Theory : Cauchy�s theorem for an open star shaped domain, Cauchy�s integral formula, Cauchy�s estimate, Taylor�s theorem, Liouville�s theorem, Morera�s theorem, Fundamental theorem of algebra.

Stagularities : Three types of isolated inequalities, Casorati-Weierstrass theorem, Laurent�s theorem. Classification of singularity by the principal part of Laurent�s expansion.

Meroniophic function, Argument principle, Rouche�s theorem.

Residue calculus : The residue Theorem and its application; evaluation of standard types of integrals by the residue calculus method.

Maximum modulus principle : Maximum modulus principle and open mapping theorem, their corollaries, automorphisms of the unit disc, harmonic functions.

Conformal mappings : Definition and examples; Riemann mapping theorem (statement only)

Harmonic functions : Mean value property, maximum and minimum principle, Dirichlet problem for the unit ball.

References :

1. J.B. Conway : Function of one complex variable, Second Edition, Narosa Publishing Company, 1980.

2. K. Knopp : Theory of functions Vol I and II, Dover.

3. W. Rudin : Real and Complex Analysis, Tata McGraw Hill II (1974).

4. L.V. Ahlfors : Complex Analysis, McGraw-Hill (International Student Edition) 1966.

5. S. Lang : Complex Analysis, Addison Wesley, 1977.

6. Shastri A.R. : Complex Analysis, Mac Millan, India, 1999.

Paper 5 � Combinatorics

Basic Counting: The sum rule and the product rule, two-way counting, permutations and combinations, Binomial and multinomial coefficients, Pascal identity, Binomial and multinomial theorems. Advanced Counting : Types of occupancy problems, distribution of distinguishable and indistinguishable objects into distinguishable and indistinguishable boxes (with conditions on distributions), Stirling numbers of second kind, Bell numbers, Stirling numbers of first kind, Partition numbers and Catalan numbers.

Inclusion-Exclusion Principle : Number of objects having at least one m properties, none of the m properties, exactly r of the m properties, applications to derangement and forbidden position problems, rook polynomials, Mobius inversion formula for partially ordered sets. Pigeon-hole principle: Pigeon-hole principle, some generalizations and applications, Erdos-Saekeres theorem on monotone subsequences..

Recurrence Relations : The Fibonacci sequence, Linear, homogeneous recurrence relations with constant coefficients; proof of the solution in case of distinct roots and statement of the theorem giving a general solution (in case of repeated roots), Iteration and Induction, Difference tables.

Generating functions and their Applications : Ordinary and exponential generating functions, algebraic manipulations with power series, generating function for counting combinations with and without repetitions, exponential generating function for Bell numbers, applications to counting, Use of generating functions for solving recurrence relations.

Partitions : Partitions of integers and their generating functions, Ferrers diagram and its uses, Conjugate partitions, Euler�s Pentagonal theorem.

Polya Theory of Enumeration : Equivalence relations and orbits under a permutation group action, Orbit-stabilizer theorem, Burnside Lemma and its applications, Cycle index, Pattern inventory, Polya�s theorems and applications.

Marriage Theorems : Systems of distinct representatives, Hall�s theorem on systems of distinct representatives, matchings in bipartite graphs and application, Birkhoff-von Neumann theorem on doubly stochastic matrices.

Discrete Probability : Sample spaces, Probability measures, Conditional probabilities and independence, Bayes theorem and its applications, Random variables, expectation and variance, examples of discrete random variables : binomial, geometric, hypergeometric and Poisson distributions, independent and repeated trials, Chebyshev inequality.

References :

1. V. Krishnamurthy, Combinatorics : Theory and applications, Affiliated East - West Press, 1993.

2. R.A. Brualdi, Introductory Combinatorics, North-Holland Publishing Company, 1978.

3. D.I.A. Cohen, Basic Techniques of Combinatorial Theory, John Wiley & Sons, 1978.

4. W. Feller, An Introduction to probability theory and its applications, Volume I, John Wiley&Sons, 1968.

5. V.K. Balakrishnan, Theory and problems of Combinatorics, Schaum�s outline series, Mc-Graw-Hill Inc., 1995.

6. A Tucker, Applied Combinatorics, John Wiley & Sons, 1980.